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No. 840.

THE RELATION OF TENSILE STRENGTH TO COMPOSITION IN STRUCTURAL STEEL— ADDITIONAL DISCUSSION ON PAPER No. 811.*

By WILLIAM R. WEBSTER, A. C. CUNNINGHAM, H. H. CAMPBELL and WILLIAM METCALF.

PRESENTED SEPTEMBER 7TH, 1898.

WILLIAM R. WEBSTER, ESQ.—Owing to absence from the country Mr. Webster. the writer's attention was not called to this interesting paper until quite recently. The author uses values of 1 000 lbs. increase for each .01% of carbon and phosphorus, which are the averages of Mr. Campbell's values for the same elements in acid and basic open-hearth steel, as is here shown.

MR. CAMPBELL'S VALUES.

	Carbon.	Phosphorus.	Average of C. & P.
Acid steel	1 250	890	1 070
Basic steel	950	1 050	1 000
Average for acid and basic.	1 100	970	1 035

Using for round numbers 1 000 lbs.

In like manner the author's base of 40 000 lbs. is an average of the bases used by Mr. Campbell for acid and basic steel, with an addition

* "The Relation of Tensile Strength to Composition in Structural Steel," by A. C. Cunningham, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. xxxviii, p. 78.

Mr. Webster.

TABLE No. 1.*—Acid Open-Hearth Steel.

Number of group.	AVERAGE ANALYSIS. PERCENTAGE OF.				Average actual ultimate, pounds per square inch.	CAMPBELL.		WEBSTER.			CUNNINGHAM.	
	Carbon by combustion.	Phosphorus.	Manganese.	Sulphur.		Calculated ultimate, pounds per square inch.	Difference.	Calculated ultimate, pounds per square inch.	Difference.	Difference—Constant.	Calculated ultimate, pounds per square inch.	Difference.
1.	.061	.069	.29	.071	54 700	52 120	— 2 580	53 300	—1 400	—3 400	53 000	— 1 700
6.	.087	.060	.33	.043	55 470	54 470	— 1 000	54 000	—1 470	—3 470	54 700	— 770
11.	.096	.075	.25	.060	57 200	56 890	— 310	56 430	— 770	—2 770	57 100	— 100
16.	.117	.056	.39	.028	56 950	57 740	+ 790	58 400	+1 450	— 550	57 300	+ 350
21.	.125	.047	.41	.032	56 960	57 910	+ 950	58 860	—1 900	— 100	57 200	+ 240
26.	.130	.053	.40	.042	59 170	59 050	— 120	60 540	+ 370	—1 680	58 300	— 870
31.	.143	.099	.39	.065	62 830	64 710	+ 1 880	69 940	—7 110	+5 110	64 200	+ 1 370
36.	.155	.069	.41	.034	60 940	63 500	+ 2 560	66 180	—5 240	+3 240	62 400	+ 1 460
41.	.170	.074	.43	.046	64 840	65 760	+ 920	68 670	—3 730	+1 730	64 400	— 440
46.	.207	.088	.41	.047	69 410	71 480	+ 2 070	73 500	+4 090	+2 090	69 500	+ 90
51.	.229	.065	.50	.082	70 810	72 090	+ 1 280	72 220	—1 410	— 590	69 400	— 1 410
56.	.424	.043	.57	.031	94 470	93 730	— 740	......			86 700	— 7 770
61.	.113	.061	.43	.038	57 140	57 700	+ 560	59 500	+2 360	+ 360	57 400	+ 260
66.	.116	.082	.50	.069	60 870	59 980	— 940	63 290	+2 420	+ 420	59 800	— 1 070
71.	.118	.075	.42	.045	59 110	59 550	+ 440	62 070	+2 960	+ 960	59 300	+ 190
76.	.119	.065	.43	.028	61 020	58 780	— 2 240	60 320	— 700	—2 700	58 400	— 2 620
81.	.134	.045	.48	.035	58 820	58 820	0	60 810	+1 990	— 10	57 900	— 920
86.	.151	.051	.64	.033	62 650	61 410	— 1 240	65 380	+2 730	+ 730	60 200	— 2 450
91.	.183	.027	.68	.030	65 100	63 150	— 1 950	64 590	— 560	—2 560	61 000	— 4 100
96.	.212	.073	.82	.039	71 870	70 750	— 1 120	75 610	+3 740	+1 740	68 500	— 3 370
101.	.242	.076	.86	.049	78 020	74 650	— 3 370	79 360	+1 340	— 660	71 800	— 6 220
106.	.374	.057	.83	.035	90 750	88 930	— 1 820	......			83 100	— 7 650
111.	.392	.029	.63	.022	98 180	88 610	— 9 570	......			82 100	—16 080
116.	.333	.041	.65	.026	87 410	82 540	— 4 870	......			77 400	—10 010
121.	.480	.082	.69	.022	111 740	99 530	—12 210	......			91 200	—20 540
126.	.555	.109	1.13	.042	123 020	115 460	— 8 100	......			106 400	—17 220

* The figures in this table are the same as those given in Table No. 1 of the original paper (*Transactions*, Am. Soc. C. E., Vol. xxxviii, p. 82), except that those under the heading WEBSTER are corrected by the writer.

for the average of .40 manganese, assumed by the author to be in this steel. The following will show how this works out:

Mr. Campbell's Values.

	Base.	Addition for Manganese.	Base Manganese.
Acid steel..............	38 600	0	38 600
Basic steel	37 430	3 400	40 830
Average for acid and basic.................	38 015	1 700	39 715

Using for round numbers 40 000 lbs.

The application of these values for carbon and phosphorus, by the author, to the cases given secures very good results in most instances. The results given as being obtained from the use of the writer's tables on this series of tests, are, however, surprising, and contain many

TABLE No. 2.*—BASIC OPEN-HEARTH STEEL.

Mr. Webster.

Number of group.	Average Analysis. Percentage of.				Average actual ultimate, pounds per square inch.	Campbell.		Webster.		Cunningham.	
	Carbon by combustion.	Phosphorus.	Manganese.	Sulphur.		Calculated ultimate, pounds per square inch.	Difference.	Calculated ultimate, pounds per square inch.	Difference.	Calculated ultimate, pounds per square inch.	Difference.
127....	.051	.008	.25	.027	46 630	45 240	−1 390	44 620	−2 010	45 900	− 730
132....	.070	.009	.36	.050	47 360	48 090	+ 730	49 430	+2 070	47 900	+ 540
137....	.074	.021	.38	.034	49 280	49 900	+ 620	50 230	+ 950	49 500	+ 220
142....	.082	.014	.43	.038	50 770	50 350	− 420	51 280	+ 510	49 600	−1 170
147....	.105	.012	.53	.082	52 970	53 170	+ 200	54 110	+1 140	51 700	−1 270
152....	.127	.010	.43	.019	52 980	54 200	+1 220	54 050	+1 070	53 700	+ 720
157....	.144	.013	.30	.037	55 260	55 080	− 230	54 790	− 470	55 700	+ 440
162....	.165	.021	.41	.051	57 220	58 800	+1 580	60 290	+2 970	58 600	+1 380
167....	.170	.040	.42	.043	61 070	61 770	+ 700	63 280	+2 210	61 000	+ 70
172....	.183	.006	.36	.080	57 350	58 710	+1 360	57 950	− 760	59 100	+1 750
177....	.194	.024	.51	.040	61 340	62 610	+1 270	63 620	+1 010	61 700	+ 360
182....	.209	.024	.47	.053	63 220	63 800	+ 580	65 160	+1 940	63 300	+ 80
187....	.220	.015	.40	.018	63 560	63 300	− 260	62 000	−1 570	63 500	+ 60
192....	.248	.018	.40	.033	66 220	66 280	+ 60	65 440	− 780	66 600	+ 380
197....	.301	.017	.65	.029	76 890	73 340	−3 550	72 130	−4 650	71 800	+5 090
202....	.052	.019	.35	.054	49 010	47 340	−1 670	48 880	− 80	47 100	+1 910
207....	.062	.015	.21	.036	48 980	46 680	−2 300	45 680	−1 050	47 700	−1 280
212....	.076	.018	.41	.062	50 890	50 080	− 850	52 010	+1 130	49 400	−1 480
217....	.084	.021	.25	.033	50 900	49 740	−1 160	48 680	−2 220	50 500	− 400
222....	.094	.046	.43	.036	54 800	54 840	+ 40	55 310	+ 510	54 000	− 800
227....	.108	.027	.44	.064	54 950	53 790	−1 160	55 610	+ 660	53 000	−1 950
232....	.117	.053	.46	.035	57 210	58 020	+ 810	59 380	+2 170	57 000	− 210
237....	.125	.036	.54	.060	58 790	57 680	−1 110	60 450	+1 660	56 100	−2 690
242....	.134	.055	.51	.036	59 110	60 270	+1 160	62 540	+3 430	58 900	− 210
247....	.144	.028	.52	.034	58 970	58 470	− 500	60 000	+1 080	57 200	−1 270
252....	.153	.012	.46	.027	58 970	57 130	−1 840	57 460	−1 510	56 500	−2 470
257....	.173	.021	.53	.021	60 810	60 570	− 240	60 890	+ 80	59 400	−1 410
262....	.190	.047	.72	.037	66 480	66 530	+ 50	68 850	+2 370	63 700	−2 780
267....	.215	.011	.42	.024	63 470	62 580	− 890	61 580	−1 790	62 600	− 870
272....	.338	.017	.62	.026	77 950	76 600	−1 350	74 640	−3 310	75 500	−2 450

* The figures in this table are the same as those given in Table No. 1 of the original paper (*Transactions*, Am. Soc. C. E., Vol. xxxviii, p. 83), except that those under the heading WEBSTER are corrected by the writer.

errors. Tables Nos. 1 and 2 herewith give the results, properly estimated by the writer's tables, and corresponding differences between estimated and actual ultimate strength. It will be seen in Table No. 1, as corrected, that the writer's results are a little too high, and that, by deducting the constant of 2 000 lbs., these results are very much improved. This constant, as may be seen by referring to the writer's papers, is one that has often been suggested for use in particular mill treatment or kind of material rolled. In this case the bars were 2 ins. x ⅜ in., rolled for small ingot.

The writer's method of investigation was one of successive approximations, and, as the results of each individual test were on separate cards, it was an easy matter to eliminate the effect of one element by placing all the tests having the same carbon in one pile; for in-

Mr. Webster. stance, .20 carbon. Thus the difference in the tensile strength in this particular lot of tests would not be due to carbon, at least so far as known at present, although its effect in the different proportions of other elements present might be greater in one case than in another. Assuming, however, that the difference was not due to carbon, the writer tried to find a value for the other elements that would fulfill the condition and meet the differences. After this, grouping in one pile, the cards having the same manganese, the direct influence of manganese was eliminated, and proceeding in same manner for phosphorus, and then for sulphur, the values used in the tables were arrived at. This, of course, took a great deal of time and hard work, but the values secured met the requirements of the conditions under which the work was done.

The author states:

"The later investigations of Mr. Campbell are the most complete and scientific of any that have yet been undertaken in this line. With 3 163 tests made upon 2 x $\frac{3}{4}$-in. test bars of a known and uniform condition, arranged in 272 groups of similar conditions as to strength and composition, Mr. Campbell has, by the method of least squares, arrived at the strengthening effect of the various components of steel," notwithstanding the fact that Mr. Campbell in his paper admits that in applying the method of least squares and using all the elements, his results were not intelligible, and that first one element and then another had to be eliminated from his equations, until he could obtain results that were applicable to the problem at hand. The writer does not, of course, know how familiar Mr. Cunningham and Mr. Campbell are with this method of least squares, but believes that the question is still an open one as to whether or not it is the best method to use.

Messrs. Cunningham's and Campbell's results are based on figures obtained by grouping the original tests in accordance with their chemical composition and taking the average results. The writer thinks that they will find, upon further investigation of the subject, that it is much better to take individual cases, and not an average chemical composition, as this tends to mask the influence of the elements when existing in different proportions. In the application of any of these values to individual cases, tests in which the estimated ultimate strengths do not agree at all with the tensile tests will be found from time to time, and from such apparently abnormal cases more will be learned of the influence of each element, and of proper treatment in heating and rolling, than from the tests which agree with the estimated ultimate strengths.

In order to compare the two methods in their application to individual tests, the writer has taken the 408 tests given in his previous papers, has worked out the estimated ultimate strength in each case by the author's figures, and tabulated the results in the same general form as used in the writer's previous work. The estimated ultimate strength has been deducted from the actual ultimate strength, and in

all cases in the following tables the writer's figures are, of course, the Mr. Webster.
same as given in former papers. In this table the differences between
the estimated and the actual ultimate strengths are subdivided as
noted in the first column, and are recorded in the proper division in
columns marked "Cunningham" and "Webster," corrections for
thickness not being applied in either case. This table shows the
necessity of considering other elements than carbon and phosphorus
in estimating the strength of steel from its chemical composition.
The tests recorded in the division marked "plus over 10 000 lbs." are
not considered in the summary, as they are included in the division of
"plus over 5 000 lbs.," and are only given to show how much out of the
way are the ultimate strengths calculated by the author's formula,
when applied to individual tests of steel under 75 000 lbs. tensile
strength.

WEBSTER'S 408 TESTS.

Differences.	Cunningham.	Webster.
+ over 10 000 lbs..........................	28	0
+ " 5 000 "	175	18
+ 4 000 to 5 000 "	53	18
+ 3 000 " 4 000 "	44	26
+ 2 000 " 3 000 "	50	35
+ 1 000 " 2 000 "	34	53
Within ± 1 000 lbs..........................	44	106
− 1 000 to 2 000 lbs..........................	5	54
− 2 000 " 3 000 "	2	41
- 3 000 " 4 000 "	1	28
− 4 000 " 5 000 "	0	18
− over 5 000 "	0	11
− " 10 000 "	0	0

SUMMARY OF ABOVE.

	Cunningham.	Webster.
Total + more than 1 000 lbs.................	356	150
" − " " 1 000 "	8	152
Difference....	+ 348	− 2
Per cent. within 1 000 lbs.................	10.8	26.0
" " 2 000 "	20.3	52.2
" " 3 000 "	33.1	70.8
" " 4 000 " .....\...............	44.1	84.1
" " 5 000 "	57.1	92.9

As Mr. Waddell[*] has compared the results in the author's Tables
Nos. 1 and 2, and has given the averages of the differences under the
headings, "Campbell," "Webster," "Cunningham," the writer, in
view of the errors that were made by the author in working out the es-
timated ultimate strengths from his tables, has made those corrections
in Mr. Waddell's tables and gives them on pages 454 and 455 in detail,
except the averages for the plus and minus differences, which he does
not understand.

Mr. Webster.

FOR ACID STEEL.

Character of differences.	AVERAGE DIFFERENCES.		
	Campbell.	Webster.	Cunningham.
Plus differences................	1 272 lbs.	2 856 lbs. 1 820 lbs.	566 lbs.
Minus differences..............	3 073 "	980 " 1 676 "	5 549 "

FOR BASIC STEEL.

Character of differences.	AVERAGE DIFFERENCES.		
	Campbell.	Webster.	Cunningham.
Plus differences...............	742 lbs.	1 583 lbs.	1 000 lbs,
Minus differences..............	1 183 "	1 684 "	1 320 "

AVERAGE FOR ACID AND BASIC STEELS.
(Mean from two preceding tables.)

Character of differences.	AVERAGE DIFFERENCES.		
	Campbell.	Webster.	Cunningham.
Plus differences...............	1 007 lbs.	2 219 lbs. 1 706 lbs.	783 lbs.
Minus differences.............	2 128 "	1 332 " 1 680 "	3 435 "

NOTE.—The figures in the second column, under the heading "Webster," are the average differences after deducting the constant of 2 000 lbs. in acid open hearth steel, as previously referred to.

FOR ACID STEEL (75 000 AND UNDER).

Character of differences.	AVERAGE DIFFERENCES.		
	Campbell.	Webster.	Cunningham.
Plus differences...............	1 272 lbs.	2 904 lbs. 1 955 lbs.	566 lbs.
Minus differences.............	1 150 "	980 " 1 778 "	1 652 "

FOR BASIC STEEL (75 000 AND UNDER).

Character of differences.	AVERAGE DIFFERENCES.		
	Campbell.	Webster.	Cunningham.
Plus differences...............	742 lbs.	1 583 lbs.	659 lbs.
Minus differences.............	1 001 "	1 223 "	1 312 "

AVERAGE FOR ACID AND BASIC STEEL (75 000 AND UNDER).   Mr. Webster.
(Mean from two preceding tables.)

Character of differences.	AVERAGE DIFFERENCES.		
	Campbell.	Webster.	Cunningham.
Plus differences................	1 007 lbs.	2 243 lbs. 1 769 lbs.	613 lbs.
Minus differences.............	1 076 "	1 101 " 1 501 "	1 482 "

NOTE.—The figures in the second column, under the heading "Webster," are the average differences after deducting the constant of 2 000 lbs. in acid open-hearth steel as previously referred to.

Mr. Metcalf* remarks that the manufacturers can vary the strength of steel by manipulation from 50% to 100% one way or the other. The writer does not consider the criticism a fair one, as it is the manufacturers of structural steel who depend more than any one else on the relation between the chemistry and the physical properties of steel.   They all apply the steel to the orders by its chemical composition, and if the tension tests of the finished product do not give the results that they expected from the analyses of the heat of steel, they at once examine into the conditions of heating and rolling; and it is by this close watch of both the chemical composition and physical treatment of the steel that they have made such great advances in its manufacture during recent years.

Mr. Metcalf fails to state that in producing the changes referred to the manufacturer would also greatly change the elastic limit, the percentage of stretch, the percentage of reduction of area, and the bending properties of the steel treated.   These changes, for any given treatment, would be greater or less, in accordance with the chemical composition of the steel so treated, and would emphasize the effects of the carbon, phosphorus, manganese, etc., on the steel.

Mr. Metcalf would not consider it unreasonable to state that, if a piece of steel with a known chemical composition, and a known treatment in heating and rolling produced, say, 65 000 lbs. tensile strength, at a future time another heat of steel with exactly the same chemical composition with the same treatment in heating and rolling will produce about the same result.   If there were enough standard tests to cover all cases of structural material, and they could be recorded properly, it would be merely a matter of turning to such records, in order to know what a given heat of steel should produce in the finished product, without having to interpolate for the cases not so recorded. The avoidance of the necessity for this interpolation is sought by giving values to each of the elements.   The problem is certainly a very difficult one and is still somewhat obscure.   Marked progress,

* *Transactions*, Am. Soc. C. E., Vol. xxxviii, p. 84.

Mr. Webster. however, has been made, inasmuch as it was only in 1892 that no less an authority than Mr. H. M. Howe, in summing up this whole matter said :

"If these views are correct, then, no matter how great and extended our knowledge of ultimate composition, and how vast the statistics on which our inferences are based, if we attempt to predict mechanical properties from them accurately, we become metallurgical Wigginses."

Mr. Cunning-
ham.
A. C. CUNNINGHAM, M. Am. Soc. C. E.—Mr. Webster's mathematical demonstration, attempting to show that the base and factors used in the original paper are the averages of those deduced by Mr. Campbell, proves that such is not the case.

These factors were arbitrarily assumed after numerous algebraic solutions, on the assumption that carbon and phosphorus were the only hardeners, and that the strength of a theoretical steel free from carbon and phosphorus would be 40 000 lbs. per square inch.

Several hundred trials of these factors and this base, on various kinds of steel made by a dozen different manufacturers, led to the conclusion that they were approximately right. Their application to Mr. Webster's steel was unsatisfactory, and in view of the good results obtained with many other steels, naturally leads to the conclusion that his analyses may be questioned.

In using Mr. Webster's tables two results could be generally obtained; the one giving the most nearly correct result was taken, but no further manipulation was attempted.

However, even if Mr. Webster's tables are incorrect, and also his assumptions that all elements have a strengthening effect on steel, his work has been of great value in drawing renewed attention to this subject and encouraging other investigations.

In the present case a rule has been sought that, while approximately correct, should also be easily applied. Expressed as an equation, this rule is as follows:

40 000 lbs. + 1 000 lbs. for every .01% of carbon + 1 000 lbs. for every .01% of phosphorus = ± tensile strength.

This rule is not only easily carried by the memory, but also can be applied mentally with little effort, and, as Mr. Webster admits, gives very good results in most instances. Its trial has not been confined to Mr. Campbell's tests, but, as Mr. Webster advises, it has been used extensively on individual tests.

The rule is intended to establish for the inspector and the engineer a relation between the analyses furnished by the manufacturer and the lists made; when it does not, it is time for investigation.

Except in special cases, the analysis furnished by manufacturers represents the average chemical composition of the steel, while the actual test represents the strength of a concrete portion. To attempt to establish an absolute relation between these quantities is like pacing the diameter of a circle and then computing the circumference to the thousandth part of an inch.

The rule given must stand or fall upon its merits, and these can only be determined by sufficient trial. When failures occur analysis of the test piece will show whether the rules or the test is at fault. Mr. Cunningham.

H. H. CAMPBELL, M. Am. Soc. C. E.—The investigation which is the subject of the paper, will be found in a book by the writer, entitled "Structural Steel." On page 284 is the following sentence: "The most comprehensive and systematic study of the physical formula of steel has been carried out by W. R. Webster." A description of his method is given and then it is shown that portions of his table contain "absolutely irreconcilable conditions, for Mr. Webster takes as his starting point the dictum that carbon is a constant, and proceeds to construct a table in which it is not a constant at all, and in which it is not even constantly irregular." This point is discussed at length and is proven by quotations from Mr. Webster's results. It was the intention of the writer to treat a co-investigator with courtesy, while exercising the right to openly discuss the faults in his theories. The book did two distinct things. It declared Mr. Webster's results inconsistent with his premises, and it offered a new system of calculation by the method of least squares. Mr. Cunningham has accepted the latter method, and has undertaken to condense the results, so that engineers may have a short and simple formula by which to calculate the strength of steel when the physical composition is known. Mr. Campbell.

It is not fair to hold the writer liable for the variations in the author's results, but he does assume all responsibility for the original formula and for the method employed. He knows of no way by which the problem under consideration may be solved save by the method of least squares. There may be a better way, but, as far as the writer knows, the only other method offered has been the system of Mr. Webster, which, as shown above, gives results that contradict the original assumptions, and hence must be without scientific standing.

The method of least squares is open to the criticism of any one, but it is respectfully submitted that the remark of Mr. Webster is not criticism. He states that he does not know " how familiar Mr. Cunningham and Mr. Campbell are with the method of least squares." This has nothing to do with the case. The question is whether the work is right and the method applicable. This may be decided by a mathematical or philosophical demonstration by any one who knows more than the author or the writer.

The original formulas, as printed in the book mentioned, have been used for the last two years to calculate the tensile strength of every heat made in the Open-Hearth Department of the Pennsylvania Steel Company; and in the case of structural steels having a tensile strength of between 50 000 and 70 000 lbs. per square inch, the mathematical results are almost always within 2 000 lbs. of the figures obtained on the testing machine. So reliable have the formulas proved that a wide

Mr. Campbell. variation (say of 5 000 lbs.) is always investigated, with the result that almost always errors are discovered in the chemical determinations, or else the conditions under which the test piece has been rolled are found to have been abnormal.

The chemical errors are almost always in the carbon content. This is one of the vital errors in Mr. Webster's system, since the calculations on individual heats necessitate using color determinations, and these are not sufficiently reliable for scientific investigations.

Under the system used by the writer and endorsed by the author, each element is determined in the most approved way, and the problem is solved by a mathematical method devised for just such conditions.

Mr. Metcalf. WILLIAM METCALF, Past President, Am. Soc. C. E. —The writer agrees fully with Mr. Webster's position, that, given a certain physical specification, a steel-maker should know whether it could be filled or not, and if it could be met, he should know just about what composition of steel to use. If an engineer were to accept such steel made to a formula, without careful physical tests, he would know little about the properties of the material he received. This is all the writer claimed in discussing the paper, and the author reminded him that it was not claimed that composition and formulas would take the place of tests. This brought the author and writer into substantial agreement, and if Mr. Webster would consider the discussion as a whole he would see that he and the writer agree also.

The writer highly appreciates the great value of the work of Messrs, Webster and Campbell. and hopes they will continue it. No matter how perfectly they may work out their questions, they will never eliminate the human element unless they rework and purify the individuals as they can their heats of steel.

Mr. Webster. WILLIAM R. WEBSTER, Esq.—Mr. Metcalf's full explanation of his former remark now puts the whole matter in such shape that it is not open to any misunderstanding. The writer never advocated accepting any material without making physical tests, but has tried to find some guide for those asking for both chemical and physical tests, in order that one may agree with the other.

The point raised by the author, "that the analysis furnished by the manufacturers represents the average chemical composition of the steel," is fully appreciated. It was for this very reason that, in the writer's investigation, analyses were made of drillings taken from the broken test pieces. There were over one thousand pieces analyzed, and in many cases the drillings taken from the same test pieces were sent to the chemist under two numbers, in order to check his work.

From this it is fair to assume that the differences referred to by the author, when the ultimate strength of the writer's 408 tests were estimated by his values, are not due to errors in chemical analyses. The whole trouble is caused no doubt by the manganese being in different

proportions for the same carbons, and as the effect of manganese is not Mr. Webster. considered, just such differences are apt to result. This can be shown by any series of individual tests; the writer merely used his tests as they were at hand.

The author's comparison of calculating the circumference of a circle to the thousandth part of an inch from its paced diameter is not a parallel case. As manganese has an effect on the ultimate strength of steel, it should always be allowed for directly instead of indirectly, by giving the carbon a larger value which holds good only when there is a fixed relation between the carbon and manganese present. The steel works now use different methods of recarburization, and as this investigation is continued it will be found necessary to allow for the effect of manganese in both acid and basic steel, whether made in the open-hearth furnace or in the Bessemer converter.

In stating that the author's values for carbon and phosphorus were the averages of Mr. Campbell's values for these elements, and that his base was the average of the bases used by Mr. Campbell for acid and basic steel, the writer did not in any way mean to detract from or reflect on the value of the work, and he is glad to hear that the values were arrived at independently. As the work confirms that of Mr. Campbell it certainly adds to the value of each, the difference being only 35 lbs. in the case of each .01% of phosphorus and carbon, and 285 lbs. in the base used.

The writer cannot understand how the author gets two results in using his values. This is impossible, as will be shown later in these remarks. Mr. Campbell is to be congratulated in that he finds such a close relation between the estimated ultimate strengths and the actual strength of the 2 x $\frac{3}{8}$-in. bars rolled from the small test ingots. No doubt the tests of the finished material give correspondingly good results, although he has not said so.

" We discovered many years ago that we had been running with an error of .11% in all our low carbon determinations, and .13 in all the high steels. Thus steel of .09 carbon had been regularly determined as .20, and .50 carbon as .63. Customers ordered steel, found it right, or found it too hard or too soft, and ordered the next lot accordingly. Years had rolled by, and every customer knew just what he wanted, and could learnedly discuss the special nature of .64 and of .76 carbon. The discovery of the error in the standards was a rude shock, and the change to the new order of things was the work of many months, and a diplomatic catering to prejudice, mixed with a very strong disinclination to an open acknowledgment that we had been altogether wrong."

This quotation from Mr. Campbell's book (page 7) is a fair illustration of how little attention was paid formerly, by manufacturers or any one else, to the relation between chemical composition and the ultimate strength of steel. Whereas, to-day some mills are rolling the steel into the finished product altogether from the estimated ultimate strength based on the chemical composition of each heat. The only

Mr. Webster. tests made are on the finished material after the whole of the heat has been rolled.   The steel is never allowed to cool from the time it is cast until it leaves the rolls as finished angles, bars, etc.   This great advance in method of working, means a yearly saving in the coal bill of thousands of dollars, and another large saving in the cost of handling the material.

Mr. Campbell's views as to the value of the method of least squares have materially changed since he wrote his "Structural Steel," as he states on page 297:

"It is with no little disappointment that I am forced to confess that further investigation throws grave doubts on the validity of this method of least squares when applied to such a number of unknown quantities, and when any one of these quantities is of very little importance."

He now states:

"Under the system used by the writer and endorsed by Mr. Cunningham, each element is determined in the most approved way, and the problem is solved by a mathematical method devised for just such cases."

Mr. Campbell's results were unintelligible until he assumed that copper, silicon and sulphur had no effect on the ultimate strength of steel.   Leaving these elements out he then obtained several results for each of the elements, in the order in which they are given below, for each class of steel.   In the latter part of his investigation he considered a new series of acid and basic steels.   This gave him for the acid steel 56 groups in the new series, as against 70 groups in the first series.   For the basic steel he had 74 groups in the first series and 72 groups in the new series.

The values he has adopted may be summarized as follows:

Mr. Campbell's Values for Pure Iron and Increase Due to .01 Per Cent. of Carbon, Phosphorus and Manganese, in Pounds per Square Inch.

In b and e for acid steels, and in g and j for basic steels, the increase due to manganese was not considered.   Mr. Campbell remarks, regarding the value of 1 444 lbs. for phosphorus in j,

"This value of phosphorus was not sustained by any other evidence."

"The factor R represents an allowance for the conditions under which the piece is rolled, whether finished hot or cold.   In the present series of groups it is zero."

ACID STEELS.

		C.	P.	Mn.	Pure Iron.		
(a)	Old series..	1 529 +	1 316 +	39 +	34 326		= Ultimate strength
(b)	Old series..	1 485 +	1 260		+ 33 000		= Ultimate strength
(c)	New series..	1 126 +	716 +	3 +	40 439		= Ultimate strength
(d)	Old series..	1 368 +	1 068 −	23 +	37 544		= Ultimate strength
(e)	Both series..	1 210 +	890		+ 38 600 +	R	= Ultimate strength

Basic Steels.

<div style="text-align:right">Mr. Webster.</div>

	C.	P.	Mn.	Pure Iron.	
(f) Old series	1 035 +	941 +	53 +	38 996	= Ultimate strength
(g) Old series	1 085 +	1 200		+ 40 000	= Ultimate strength
(h) New series	935 +	939 +	114 +	36 335	= Ultimate strength
(i) Old series	1 035 +	941 +	53 +	38 996	= Ultimate strength
(j) Both series	998 +	1 444		+ 39 987	= Ultimate strength
(k) Both series	950 +	1 050 +	85 +	37 430 + R	= Ultimate strength

Mr. Campbell estimated the ultimate strength of each of his groups by several of the above values, and the difference between the estimated ultimate strengths and the average actual ultimate strengths of the groups were small in most cases for each class of steel, but the best results were obtained by the last values given in the above tables.

The value of any one of the elements, carbon, phosphorus or manganese, and the value of pure iron, seems to depend on the number of tests considered. If the effect of each element is constant and not dependent on the amount of other elements present, this should not be the case. There seems to be a disturbing element, running all through the successive values arrived at, which strongly indicates that the values of carbon, phosphorus and manganese are not constant under all conditions of the series of tests in either the acid or the basic steels considered.

The writer does not consider that the results obtained justify Mr. Campbell in taking the strong stand that manganese does not affect the ultimate strength of acid open-hearth steel until it is raised to over .60 per cent. In this class of steel he uses 1 210 lbs. for each .01% of carbon, instead of 950 lbs. as in the basic steel. This increase of 260 lbs. for each .01% of carbon represents an increase of 27% in its value, and it includes the effect of the manganese and part of the effect of the phosphorus. This is indicated by the lower value of 890 lbs. instead of 1 050 lbs. for phosphorus, being a reduction of 160 lbs. for each .01%, or a reduction of 15%, and the manganese is not considered at all, being a reduction of 85 lbs. for each .01% of that element.

The accuracy of Mr. Campbell's work in using the method of least squares in arriving at these values is not disputed, but his results indicate strongly that the amounts of the elements present have an indirect effect on each other, and that this is shown by the different values arrived at for each element. If Mr. Campbell's position is correct, in that the method of least squares would not give intelligible results, when he considered the effect of copper, silicon and sulphur with the other elements, on account of the copper, silicon and sulphur having very little effect on the ultimate strength of the steel, it certainly casts a reflection on the applicability of this method to the problem in hand,

Mr. Webster. as it presupposes a knowledge of the effects of three of the unknown quantities under consideration.

In his investigation the writer found that the indications were that the effect of carbon was 800 lbs. per .01% and that of phosphorus 800 lbs. per .01%, when in the presence of .06, .07 and .08% carbon, but, as the carbon increased, the effect of phosphorus increased up to .15% carbon, where .01% phosphorus equaled 1 500 lbs. per square inch. This is shown in Table No. 3.

TABLE No. 3.—WEBSTER'S VALUES FOR PHOSPHORUS IN THE PRESENCE OF .05 TO .15 PER CENT. CARBON, INCLUSIVE.

Per Cent.	Carbon .05, .06, .07, .08.	Carbon .09	Carbon .10	Carbon .11	Carbon .12	Carbon .13	Carbon .14	Carbon .15 and over.
P .00...	0	0	0	0	0	0	0	0
" .005 .	400	450	500	550	600	650	700	750
" .01...	800	900	1 000	1 100	1 200	1 300	1 400	1 500
" .015..	1 200	1 350	1 500	1 650	1 800	1 950	2 100	2 250
" .02...	1 600	1 800	2 000	2 200	2 400	2 600	2 800	3 000
" .025..	2 000	2 250	2 500	2 750	3 000	3 250	3 500	3 750
" .03...	2 400	2 700	3 000	3 300	3 600	3 900	4 200	4 500
" .035..	2 800	3 150	3 500	3 850	4 200	4 550	4 900	5 250
" .04...	3 200	3 600	4 000	4 400	4 800	5 200	5 600	6 000
" .045..	3 600	4 050	4 500	4 950	5 400	5 850	6 300	6 750
" .05...	4 000	4 500	5 000	5 500	6 000	6 500	7 000	7 500
" .055..	4 400	4 950	5 500	6 050	6 600	7 150	7 700	8 250
" .06...	4 800	5 400	6 000	6 600	7 200	7 800	8 400	9 000
" .065..	5 200	5 850	6 500	7 150	7 800	8 450	9 100	9 750
" .07...	5 600	6 300	7 000	7 700	8 400	9 100	9 800	10 500
" .075..	6 000	6 750	7 500	8 250	9 000	9 750	10 500	11 250
" .08...	6 400	7 200	8 000	8 800	9 600	10 400	11 200	12 000
" .085..	6 800	7 650	8 500	9 350	10 200	11 050	11 900	12 750
" .09...	7 200	8 100	9 000	9 900	10 800	11 700	12 600	13 500
" .095..	7 600	8 550	9 500	10 450	11 400	12 350	13 300	14 250
" .10...	8 000	9 000	10 000	11 000	12 000	13 000	14 000	15 000
.001 P =	80 lbs.	90 lbs.	100 lbs.	110 lbs.	120 lbs.	130 lbs.	140 lbs.	150 lbs.

In regard to manganese the effect did not seem to be constant per unit.*

"The effect per unit of manganese seems to decrease as the percentage of this element increases. For instance, steels of 0.20 and 0.30 Mn. show greater difference in ultimate strength than steels of 0.50 to 0.60 Mn., all other elements being the same. I have endeavored to cover this point irrespective of the percentage of carbon or phos-

* "Observations on the Relations between the Chemical Constitution and Physical Character of Steel," by William R. Webster, *Transactions, American Institute of Mining Engineers*, Vol. xxi, 1892-3, p. 767.

phorus, but upon further investigation it may be necessary to take one Mr. Webster.
or both of these elements into account in estimating the effect of high
and low manganese.

"Assuming the first addition of 0.15% manganese to increase the
ultimate strength 3 600 lbs, we have:

"Increase in Ultimate Strength by Successive Increments of Manganese.

"Manganese, per cent.		Increase in ultimate strength.	Total increase in ultimate strength from 0 Manganese.
From	To	Lbs. per square inch.	Lbs. per square inch.
0.00	0.15	3 600	3 600
0.15	0.20	1 200	4 800
0.20	0.25	1 100	5 900
0.25	0.30	1 000	6 900
0.30	0.35	900	7 800
0.35	0.40	800	8 600
0.40	0.45	700	9 300
0.45	0.50	600	9 900
0.50	0.55	500	10 400
0.55	0.60	500	10 900
0.60	0.65	500	11 400 "

When sulphur was not considered as increasing the ultimate strength
of steel, the writer found that the indications were that the ultimate
strength of pure iron without carbon, phosphorus or silicon would be
about 38 000 lbs. per square inch. To this he added the value of car-
bon and phosphorus, and gave the results in tabulated form for con-
venience in using, to estimate ultimate strength (see Table No. 4).

On the top line of Table No. 4, opposite .000 phosphorus, is given
the strength due to pure iron, and an addition of 800 lbs. for each
.01% carbon ; as, for instance, under .10 carbon, we have 38 000 plus
10 × 800 equals 46 000. Then on the other horizontal lines, opposite
.01 to .10% phosphorus, the additions have been made for the values of
phosphorus given in Table No. 3. For instance, .06% of phosphorus
with .06, .07 or .08% carbon 4 800 lbs. are added, but with .06% of
phosphorus with 11% carbon 6 600 lbs. are added, and with .15%
carbon 9 000 lbs. are added for the same amount of phosphorus.

"In basic open-hearth steel, we have deducted 2 100 lbs. from the
estimated ultimate strength; this has given fair results, but the amount
of deduction may have to be modified in using the new table.

"From the results obtained. I believe that I am safe in saying that
in all rolled steel the quality depends on the size of the bloom, or in-
got, from which it is rolled, the work put on it, the temperature at
which it is finished, and the chemical composition of the steel ; that is,
a table of this kind could be used for beams, angles, bars, etc. For
instance, a 6 x 6 x ⅜-in. angle, with a given chemical composition,
might give 4 000 pounds higher ultimate strength than indicated by
my table ; but by making this allowance, the table could be used to
advantage to show what ultimate strength another heat of steel with
different chemical composition would give if rolled into the same sized
angle. I trust that this point is clear, and that some of the shape
mills will take the matter up and let us hear from them." *

* W. R. W., *Transactions*, A. I. M. E., 1898, Vol. xxiii. p. 115.

Mr. Webster.

TABLE No. 4.—Estimated Ultimate Strengths by Webster's Values.

Base = 38 000 lbs. per square inch for iron.

Increase of 800 lbs. for each .01 carbon.

" 800 lbs. to 1 500 lbs. for each .01 phosphorus, as explained.

Carbon.	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25
Phos. .000...	43 800	43 600	44 400	45 200	46 000	46 800	47 600	48 400	49 200	50 000	50 800	51 600	52 400	53 200	54 000	54 800	55 600	56 400	57 200	58 000
" .005...	43 200	44 000	44 000	45 650	46 500	47 350	48 200	49 050	49 900	50 750	51 550	52 350	53 150	53 600	54 750	55 550	56 350	57 150	57 950	58 750
.01...	43 600	44 400	45 200	46 000	47 000	47 900	48 800	49 700	50 600	51 500	52 300	53 100	53 900	54 700	55 500	56 300	57 100	57 900	58 700	59 500
.015...	44 000	44 800	45 600	46 550	47 500	48 450	49 400	50 350	51 300	52 250	53 050	53 800	54 650	55 450	56 250	57 050	57 850	58 650	59 450	60 250
.02...	44 400	45 200	46 000	46 900	47 800	48 700	49 600	50 500	51 400	52 300	53 200	54 000	54 800	55 600	56 400	57 200	58 000	58 800	59 600	60 500
.025...	44 800	45 600	46 400	47 450	48 500	49 550	50 000	51 050	52 100	53 150	53 850	54 550	55 350	56 050	56 850	57 650	58 350	59 150	59 950	60 750
.03...	45 200	46 000	46 800	47 900	49 000	50 100	51 200	52 300	53 400	54 000	55 000	56 100	56 900	57 700	58 500	59 300	60 100	60 900	61 700	62 500
.035...	45 600	46 400	47 200	48 350	49 500	50 650	51 000	52 980	54 100	55 250	56 050	56 850	57 650	58 450	59 250	60 050	60 850	61 650	62 450	63 250
.04...	46 000	46 800	47 600	48 900	50 000	51 300	52 400	53 600	54 800	56 000	56 800	57 600	56 900	59 700	58 500	60 000	61 000	62 000	63 000	64 000
.045...	46 400	47 200	48 000	48 950	50 500	51 750	53 000	54 250	54 800	56 750	57 550	58 350	59 150	59 950	60 750	61 55"	62 350	63 150	63 950	64 750
.05...	46 800	47 600	48 400	49 700	51 000	52 300	53 600	54 600	56 200	57 000	58 300	59 100	59 900	60 700	61 500	62 300	63 100	63 900	64 700	65 500
.055...	47 200	48 000	48 800	50 150	51 500	52 850	54 200	55 550	56 500	57 750	59 050	59 850	60 650	61 450	62 250	63 050	63 850	64 650	65 450	66 250
.06...	47 600	48 400	49 200	50 500	52 000	53 400	54 600	56 000	57 400	58 000	59 300	60 100	60 900	60 700	61 500	62 900	63 900	64 900	65 900	67 000
.065...	48 000	48 800	49 600	51 050	52 500	53 950	55 400	56 850	57 800	58 750	60 050	60 850	62 150	62 950	63 750	64 550	65 350	66 150	66 950	67 750
.07...	48 400	49 200	50 000	51 500	53 000	54 500	56 000	57 500	59 000	59 500	61 300	62 100	62 900	63 700	64 500	65 300	64 600	66 900	67 700	68 500
.075...	48 800	49 600	50 400	51 950	53 500	55 050	56 600	58 150	59 700	61 250	62 050	62 850	63 650	64 450	65 250	66 05/	66 850	65 850	68 450	68 250
.08...	49 200	50 000	50 800	52 400	54 000	55 600	57 200	58 800	60 400	62 000	62 800	63 600	64 400	65 200	66 000	66 800	67 600	68 400	69 200	70 000
.085...	49 600	50 400	51 200	52 850	54 500	56 150	57 800	59 450	61 100	62 750	63 550	64 350	65 150	65 950	66 750	67 550	68 350	69 150	69 950	70 750
.09...	50 000	50 800	51 600	53 300	55 000	56 700	58 400	60 100	61 800	63 500	64 300	65 100	65 900	66 700	67 500	68 300	69 100	69 900	70 700	71 500
.095...	50 400	51 200	52 000	53 750	55 500	57 250	59 000	60 750	62 500	64 250	65 050	65 850	66 650	67 450	68 250	69 050	69 850	70 650	71 450	72 250
.10...	50 800	51 600	52 400	54 200	56 000	57 800	59 600	61 400	63 200	65 000	65 800	66 600	67 400	68 200	69 000	69 800	70 600	71 400	72 200	73 000
.001 Phos. =	80 lbs	80 lbs	80 lbs	90 lbs	100 lbs	110 lbs	120 lbs	130 lbs	140 lbs	150	150 lbs	150 lbs	150 lbs	150 lbs	150 lbs	150 lbs	150 lbs	150 lbs	150 lbs	150 lbs

"When rolling heavy steel plates, trouble is often caused by finish- Mr. Webster. ing them at too high a temperature, which gives a material with crystalline fracture, poor reduction of area, and poor bends. In order to guard against this and control the finishing temperature, we use very light draughts in rolling, and produce as good results in heavy plates as in light ones. Too much importance cannot be given to the heat-treatment of steel. Mr. H. M. Howe's recent experiments on this subject are of the greatest value, and it is to be hoped that they will be continued on a larger scale in connection with the work of rolling and forging."*

Mr. Campbell's criticism relates more to the arrangement in tabulated form of the values arrived at by the writer for each element than to the values themselves. He, of course, had the option of using these values in either form, and one or two trials would have shown that each gives the same results, and that the tables save time and work.

In order to answer Mr. Campbell in detail, it is necessary to give his criticism in full. He states in his "Structural Steel," on pages 284, 285 and 286 :

"The most comprehensive and systematic study of the physical formula of steel has been carried out by W. R. Webster. He has used the long and laborious method of successive approximations, and, by cutting and trying, has found the effect of each element upon the ultimate strength, as well as the effect of the thickness and finishing temperature. The results are given by him as follows."

He then refers to the writer's values as already given, and continues as follows:

"An examination of these figures reveals two absolutely irreconcilable conditions, for Mr. Webster takes as his starting point the dictum that carbon is a constant, and proceeds to construct a table in which it is not a constant at all, and in which it is not even constantly irregular. By his own calculation a steel of .06% phosphorus and .10% of carbon is strengthened 1 400 lbs. by the addition of .01 % of carbon, while with .10% phosphorus it is strengthened 1 800 lbs. by the same addition. Assuredly, this is not a constant effect. Moreover, carbon does not even have a constant effect with the same content of other metalloids, for, with .10% of phosphorus, an increase in carbon from .07 to .08% raises the strength 800 lbs., while an increase from .08 to .09% strengthens it 1 800 lbs.

"It would be just as correct to conclude from these results that phosphorus is a constant and carbon a variable, as to say that carbon is a constant and phosphorus a variable. The changing values which it would be necessary to assign to carbon to fulfill the first assumption would be no more arbitrary and hypothetical than the changing values assigned to phosphorus by Mr. Webster, or the changing values which he has assigned to manganese. Thus the table which has been given is entirely indecisive, since it can be translated into two diametrically opposite readings, and it must be acknowledged that one empirical formula is as good as another, provided the same answers are obtained from both.

"This curious contradiction of the premises by the conclusion can only arise from some erroneous hypothesis in the values assigned to the different elements, for in the construction of such equations it is plain that an error in one factor must be atoned for by an opposite and

* W. R. W., *Journal of the Iron and Steel Institute*, 1894, No. 1, p. 335.

Mr. Webster. equal error in another factor. If this reasoning be true, then very little faith can be attached to the formula as an expression of fundamental laws, however accurately the mathematical results may coincide with observations.

"It is to be regretted that the earnest endeavor of Mr. Webster to write the physical formula should have been hampered by the necessity of working on sheared plates, which are finished under greater variations of temperature than angles or bars, and furthermore that these plates were of basic Bessemer steel, a material which would not be chosen for its regularity. By correcting for thickness and finishing temperature, Mr. Webster has shown that about 90% of the heats investigated came within 5 000 lbs. per square inch of what his equation calls for.

"This is a very satisfactory result, and it is not in a spirit of hypercriticism (for my own results, to be given later, display examples of the same character), but from a strictly scientific point of view, that attention is called to the very unpleasant corollary that one charge out of every ten does not give results within 5 000 lbs. Some of these undoubtedly are vitiated by wrong chemical determinations, for the carbon was determined by color, and this gives only approximate results; on others there might well be an error in estimating the finishing temperature; on others there would be mistakes in measuring and testing; while some pieces, perhaps, did actually show those peculiarities which we call abnormal, which are ascribed sometimes to oxide of iron, sometimes to nitrogen, and not infrequently to the devil, but which grow less numerous as we learn more of our art.

"I cannot believe that the complicated formula of Mr. Webster represents actual conditions, and the remainder of this chapter will attempt to show that a reasonably accurate empirical equation of steel may be written without the introduction of such manifold variations, and by the use of constant values for each element within the limits usually obtaining in structural metal. It will also be shown that the first increments of manganese do not add greatly to the strength of steel, since low-manganese metal is stronger than would be indicated by a formula that applies to steels containing higher percentages of this element."

Mr. Campbell ignores the facts, that much of the writer's work was on universal mill plate, that many heats of open-hearth steel were included in the investigation, and that the chemist's work was checked from time to time by sending him duplicate sets of drillings under different numbers. The color carbon determinations were used as is the general practice at Steelton and other works.

It is hardly necessary to work out the cases referred to by Mr. Campbell after the full explanations already given as to how the writer's tables were constituted, but they are as follows:

Percentage of C.	Percentage of P.	Pure Iron.	Additions for carbon.	Additions for phosphorus.		Difference.
.10	.06	38 000	$+ 800 \times 10 +$	$1\ 000 \times 06$	$= 52\ 000$	.....
.11	.06	38 000	$+ 800 \times 11 +$	$1\ 100 \times 06$	$= 53\ 400$	1 400
.10	.10	38 000	$+ 800 \times 10 +$	$1\ 000 \times 10$	$= 56\ 000$	.....
.11	.10	38 000	$+ 800 \times 11 +$	$1\ 100 \times 10$	$= 57\ 800$	1 800
.07	.10	38 000	$+ 800 \times 07 +$	800×10	$= 51\ 600$	.....
.08	.10	38 000	$+ 800 \times 08 +$	800×10	$= 52\ 400$	800
.08	.10	38 000	$+ 800 \times 08 +$	800×10	$= 52\ 400$	.....
.09	.10	38 000	$+ 800 \times 09 +$	900×10	$= 54\ 200$	1 800

In each of these cases the difference in ultimate strength due Mr. Webster. to .01% carbon is 800 lbs., but to this is to be added the difference due to phosphorus in the presence of the higher carbons, which, in the first case amounts to 600 lbs., in the second to 1 000, while in the third there is no difference, as the carbons are .08% and below. In the last case the difference is 1 000 lbs. This gives a total difference due to carbon and phosphorus of 1 400, 1 800, 800 and 1 800, respectively. Now, reference to Table No. 4 will show the same strength due to pure iron, carbon and phosphorus, with the same difference as shown above.

There is nothing in all this that is "absolutely irreconcilable," nor is there a case where the "results contradict the premises." The values used are those which gave the best results after many trials on over 1 000 test pieces used in this investigation, besides hundreds of trials on the steel graded in the course of the routine every-day work at the mill.

As Mr. Campbell does not believe that the writer's tables represent actual conditions, the writer would ask him if any one of the elements had a different effect per unit, due to a larger or smaller amount of the other elements present, what value would be arrived at by the method of least squares? Would it not be the average value of this element? If this is the case, why is he so positive that the writer's values are not right, and that the effect of phosphorus is not greater as the carbon increases? Also, that the effects of the other elements are always the same per unit, no matter in what amount they may be present in the steels under consideration.

In other portions of his book Mr. Campbell is not so positive in his statements on this point and others referred to above, as will be seen by the following:

"It would seem, therefore, that the regularly increasing baneful-ness of phosphorus as the carbon is raised does not portray any change in nature, but that although the effect of the metalloid in lower steels is obscured, its character is the same. No line can be drawn that can be called the limit of safety, since no practical test has ever been devised which completely represents the effect of incessant tremor. For common structural material the critical content has been placed at 10% by general consent, but this is altogether too high for railroad bridge work. All that can be said is that safety increases as phosphorus decreases, and the engineer may calculate just how much he is willing to pay for greater protection from accident.   *   *   *

"It is certain that carbon increases the strength of steel when present in small proportions, but that after a certain content is reached (say about 1.00%) there is no increase in cohesive power from a further addition. It will also be granted that this point is not a sudden break in the line, but that the effect of each unit of carbon decreases as it is approached. If this relation holds good throughout the whole series of alloys, then each successive increment of carbon will have a less effect from the starting point of pure iron.

"It is also possible, for the same reasons, that every other metalloid will follow the same rule, so that the influence of each separate alloyed

Mr. Webster.   TABLE No. 7—COMPARISON OF ESTIMATED ULTIMATE STRENGTHS,
In this Table Manga-

	Carbon =	.06	.07	.08	.09	.10	.11	.12	.13	.14
Phos. .000...	Webster............	49 300	50 100	50 900	51 700	52 500	53 300	54 100	54 900	55 700
" " ...	Campbell....basic.	46 530	47 480	48 420	49 380	50 330	51 280	52 280	53 180	54 130
" " ...	Campbell..... acid.	45 860	47 070	48 280	49 490	50 700	51 910	53 120	54 330	55 540
" " ...	Cunningham........	46 000	47 000	48 000	49 000	50 000	51 000	52 000	53 000	54 000
" .01...	Webster............	50 100	50 900	51 700	52 600	53 500	54 400	55 300	56 200	57 100
" " ...	Campbell....basic.	47 580	48 530	49 480	50 430	51 380	52 330	53 280	54 230	55 180
" " ...	Campbell..... acid.	46 750	47 960	49 170	50 380	51 590	52 800	54 010	55 220	56 430
" " ...	Cunningham........	47 000	48 000	49 000	50 000	51 000	52 000	53 000	54 000	55 000
" .02...	Webster............	50 900	51 700	52 500	53 500	54 500	55 500	56 500	57 500	58 500
" " ...	Campbell....basic.	48 630	49 580	50 530	51 480	52 430	53 380	54 330	55 280	56 230
" " ...	Campbell..... acid.	47 640	48 860	50 060	51 270	52 480	53 690	54 900	56 110	57 320
" " ...	Cunningham........	48 000	49 000	50 000	51 000	52 000	53 000	54 000	55 000	56 000
" .03...	Webster............	51 700	52 500	53 300	54 400	55 500	56 600	57 700	58 800	59 900
" " ...	Campbell....basic.	49 680	50 630	51 580	52 530	53 480	54 430	55 380	56 330	57 280
" " ...	Campbell..... acid.	48 530	49 740	50 950	52 160	53 370	54 580	55 790	57 000	58 210
" " ...	Cunningham........	49 000	50 000	51 000	52 000	53 000	54 000	55 000	56 000	57 000
" .04...	Webster............	52 500	53 300	54 100	55 300	56 500	57 700	58 900	60 100	61 300
" " ...	Campbell....basic.	50 730	51 680	52 630	53 580	54 530	55 480	56 430	57 380	58 330
" " ...	Campbell..... acid.	49 420	50 630	51 840	53 050	54 260	55 470	56 680	57 890	59 100
" " ...	Cunningham........	50 000	51 000	52 000	53 000	54 000	55 000	56 000	57 000	58 000
" .05...	Webster............	53 300	54 100	54 900	56 200	57 500	58 800	60 100	61 400	62 700
" " ...	Campbell....basic.	51 780	52 730	53 680	54 630	55 580	56 530	57 480	58 430	59 380
" " ...	Campbell..... acid.	50 310	51 520	52 730	53 940	55 150	56 360	57 570	58 780	59 990
" " ...	Cunningham......	51 000	52 000	53 000	54 000	55 000	56 000	57 000	58 000	59 000
" .06...	Webster............	54 100	54 900	55 700	57 100	58 500	59 500	61 300	62 700	64 100
" " ...	Campbell....basic.	52 830	53 780	54 730	55 680	56 630	57 580	58 530	59 480	60 430
" " ...	Campbell..... acid.	51 200	52 410	53 620	54 830	56 040	57 250	58 460	59 670	60 880
" " ...	Cunningham........	52 000	53 000	54 000	55 000	56 000	57 000	58 000	59 000	60 000
" .07...	Webster............	54 900	55 700	56 500	58 000	59 500	61 000	62 500	64 000	65 500
" " ...	Campbell....basic.	53 880	54 830	55 780	56 730	57 680	58 630	59 580	60 530	61 480
" " ...	Campbell..... acid.	52 090	53 300	54 510	55 720	56 930	58 140	59 350	60 560	61 770
" " ...	Cunningham........	53 000	54 000	55 000	56 000	57 000	58 000	59 000	60 000	61 000
" .08...	Webster............	55 700	56 500	57 300	58 900	60 500	62 100	63 700	65 300	66 900
" " ...	Campbell....basic.	54 980	55 880	56 830	57 780	58 730	59 680	60 630	61 580	62 530
" " ...	Campbell..... acid.	52 980	54 190	55 400	56 610	57 820	59 030	60 240	61 450	62 660
" " ...	Cunningham........	54 000	55 000	56 000	57 000	58 000	59 000	60 000	61 000	62 000
" .09...	Webster............	56 500	57 300	58 100	59 800	61 500	63 200	64 900	66 600	68 300
" " ...	Campbell....basic.	55 980	56 930	57 880	58 830	59 780	60 730	61 680	62 630	63 580
" " ...	Campbell..... acid.	53 870	55 080	56 290	57 500	58 710	59 920	61 130	62 340	63 550
" " ...	Cunningham........	55 000	56 000	57 000	58 000	59 000	60 000	61 000	62 000	63 000
" .10...	Webster............	57 300	58 100	58 900	60 700	62 500	64 300	66 100	67 900	69 700
" " ...	Campbell....basic.	57 080	57 980	58 930	59 880	60 830	61 780	62 730	63 680	64 630
" " ...	Campbell..... acid.	54 760	55 970	57 180	58 390	59 600	60 810	62 020	63 230	64 440
" " ...	Cunningham........	56 000	57 000	58 000	59 000	60 000	61 000	62 000	63 000	64 000
.001 Phos. =	Webster............	80	80	80	90	100	110	120	130	140
" " =	Campbell....basic.	105	105	105	105	105	105	105	105	105
" " =	Campbell..... acid.	89	89	89	89	89	89	89	89	89
" " =	Cunningham........	100	100	100	100	100	100	100	100	100

BY METHODS OF WEBSTER, CAMPBELL AND CUNNINGHAM.   Mr. Webster..
nese is .40 in all cases.

.15	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28
56 500	57 300	58 100	5ь 900	59 700	60 500	61 300	62 100	62 900	63 700	64 500	65 300	66 100	66 900
55 080	56 030	56 980	57 380	58 880	59 830	60 780	61 730	62 080	63 630	64 580	65 530	66 480	67 430
56 750	57 960	59 170	60 380	61 590	62 800	64 010	65 220	66 430	67 640	68 850	70 060	71 270	72 480
55 000	56 000	57 000	58 000	59 000	60 000	61 000	62 000	63 000	64 000	65 000	66 000	67 000	68 000
58 000	58 800	59 600	60 400	61 200	62 000	62 800	63 600	64 400	65 200	66 000	66 800	67 600	68 400
56 130	57 080	58 030	58 980	59 930	60 880	61 830	62 780	63 730	64 680	65 630	66 580	67 530	68 480
57 640	58 850	60 060	61 270	62 480	63 690	64 900	66 110	67 320	68 530	69 740	70 950	72 160	73 370
56 000	57 000	58 000	59 000	60 000	61 000	62 000	63 000	64 000	65 000	66 000	67 000	68 000	69 000
59 500	60 300	61 100	61 900	62 700	63 500	64 300	65 100	65 900	66 700	67 500	68 300	69 100	69 900
57 180	58 130	59 080	60 030	60 980	61 930	62 880	63 830	64 780	65 730	66 680	67 630	68 580	69 530
58 530	59 740	60 950	62 160	63 370	64 580	65 790	67 000	68 210	69 420	70 630	71 840	73 050	74 260
57 000	58 000	59 000	60 000	61 000	62 000	63 000	64 000	65 000	66 000	67 000	68 000	69 000	70 000
61 000	61 800	62 600	63 400	64 200	65 000	65 800	66 600	67 400	68 200	69 000	69 800	70 600	71 400
58 230	59 180	60 130	61 080	62 030	62 980	63 930	64 880	65 830	66 780	67 730	68 680	69 630	70 580
59 420	60 630	61 840	63 050	64 260	65 470	66 680	67 890	69 100	70 310	71 520	72 730	73 940	75 150
58 000	59 000	60 000	61 000	62 000	63 000	64 000	65 000	66 000	67 000	68 000	69 000	70 000	71 000
62 500	63 300	64 100	64 900	65 700	66 500	67 300	68 100	68 900	69 700	70 500	71 300	72 100	72 900
59 280	60 230	61 180	62 130	63 080	64 030	64 980	65 930	66 880	67 830	68 780	69 730	70 680	71 630
60 310	61 520	62 730	63 940	65 150	66 360	67 570	68 780	69 990	71 200	72 410	73 620	74 830	76 040
59 000	60 000	61 000	62 000	63 000	64 000	65 000	66 000	67 000	68 000	69 000	70 000	71 000	72 000
64 000	64 800	65 600	66 400	67 200	68 000	68 800	69 600	70 400	71 200	72 000	72 800	73 600	74 440
60 330	61 283	62 230	63 180	64 130	65 080	66 030	66 980	67 930	68 880	69 830	70 780	71 730	72 680
61 200	62 410	63 620	64 830	66 040	67 250	68 460	69 670	70 880	72 090	73 300	74 510	75 720	76 930
60 000	61 000	62 000	63 000	64 000	65 000	66 000	67 000	68 000	69 000	70 000	71 000	72 000	73 000
65 500	66 300	67 100	67 900	68 700	69 500	69 300	71 100	71 900	72 700	73 500	74 300	75 100	75 900
61 380	62 330	63 280	64 230	65 180	66 130	67 080	68 030	68 980	69 930	70 880	71 830	72 780	73 730
62 090	63 300	64 510	65 720	66 930	68 140	69 350	70 560	71 770	72 980	74 190	75 400	76 610	77 820
61 000	62 000	63 000	64 000	65 000	66 000	67 000	68 000	69 000	70 000	71 000	72 000	73 000	74 000
67 000	67 800	68 600	69 400	70 200	71 000	71 800	72 600	73 400	74 200	75 000	75 800	76 600	77 400
62 430	63 380	64 330	65 280	66 230	67 180	68 130	69 080	70 030	70 980	71 930	72 880	73 830	74 780
62 980	64 190	65 400	66 610	67 820	69 030	70 240	71 450	72 660	73 870	75 080	76 290	77 500	78 710
62 000	63 000	64 000	65 000	66 000	67 000	68 000	69 000	70 000	71 000	72 000	73 000	74 000	75 000
68 500	69 300	70 100	70 900	71 700	72 500	72 300	74 100	74 900	75 700	76 500	77 300	78 100	78 900
63 480	64 430	65 380	66 330	67 280	68 230	69 180	70 130	71 080	72 030	72 930	73 930	74 880	75 830
63 870	65 080	66 290	67 500	68 710	69 920	71 130	72 340	73 550	74 760	75 970	77 180	78 390	79 600
63 000	64 000	65 000	66 000	67 000	68 000	69 000	70 000	71 000	72 000	73 000	74 000	75 000	76 000
70 000	70 800	71 600	72 400	73 200	74 000	74 800	75 600	76 400	77 200	78 000	78 800	79 600	80 400
64 530	65 480	66 430	67 380	68 330	69 280	70 230	71 180	72 130	73 080	74 030	74 980	75 930	76 880
64 760	65 970	67 180	68 390	69 600	70 810	72 020	73 230	74 440	75 650	76 860	78 070	79 280	80 490
64 000	65 000	66 000	67 000	68 000	69 000	70 000	71 000	72 000	73 000	74 000	75 000	76 000	77 000
71 500	72 300	73 100	73 900	74 700	75 500	76 300	77 100	77 900	78 700	79 500	80 300	81 100	81 900
65 580	66 530	67 480	68 430	69 380	70 330	71 280	72 230	73 180	74 130	75 080	76 030	76 980	77 930
65 650	66 860	68 070	69 280	70 490	71 700	72 910	74 120	75 330	76 540	77 750	78 960	80 170	81 380
65 000	66 000	67 000	68 000	69 000	70 000	71 000	72 000	73 000	74 000	75 000	76 000	77 000	78 000
150	150	150	150	150	150	150	150	150	150	150	150	150	150
105	105	105	105	105	105	105	105	105	105	105	105	105	105
89	89	89	89	89	89	89	89	89	89	89	89	89	89
100	100	100	100	100	100	110	100	100	100	100	100	100	100

Mr. Webster. element will be represented by a curve.   This may be an arc of a circle, or a parabola, or a cycloid, or a broken line; it may be different in degree or different in nature in the case of each element; and it may vary in degree or even in nature with changes in the proportions of the associated elements.   But it will be assumed in this investigation that, within the narrow limits of the divisions of the table, the effect of a regular increase in the percentage of each metalloid would be represented by a straight line.   In other words, that an increase of carbon from .20 to .21% gives the same increment in strength as an increase from .10 to .11 per cent."

In order to study more closely the relations between the estimated ultimate strengths arrived at by the writer's method and those of Messrs. Campbell and Cunningham, the tables have been constructed on the same general plan as Table No. 5, giving the values of all three observers on each table, the carbons being in all cases from .06 to .28% inclusive.   The values are for open-hearth steel, and sulphur has not been considered, but it can be used in connection with these tables by adding 500 lbs. for each .01% of sulphur above .065% sulphur, and deducting from the values given 500 lbs. for each point below this amount.   In Table No. 5* the manganese is zero in all cases, and for that reason the values for Mr. Campbell's acid steel and Mr. Cunningham's values are higher than the others, as in these cases the value of manganese has been indirectly included in that of carbon.   Taking, however, Tables Nos. 6, 7, 8 and 9,* with manganese 30, .40, .50 and .60, respectively, the estimated ultimate strengths of one observer agree with those of the other much more closely than would be expected from the different values used for each element.   Tables Nos. 10 to 16*, inclusive, are for carbons; 29 to 51, inclusive, with manganese .0, .50, .60, .70, .80, .90, and 1 per cent.   In all other respects they are the same as Tables Nos. 5 to 9.   Of course, with the higher carbons these values may not apply, but the sooner they are tried, the sooner will the required modifications be known.

The writer believes that no one has as yet arrived at the right values for the different elements, and doubts if we will ever have a simple mathematical formula to express these values, as the elements present have, no doubt, an effect upon each other.   It will, of course, take much time and work to decide on the best modifications of values and methods now in use.

The writer has quoted freely from a paper written for presentation before the American Institute of Mining Engineers, which gives a general summary of the work of many investigators on this problem, and is a more complete answer to some of the points raised in this discussion.

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* These tables, 5 to 16 inclusive, are filed in the Library of the Society for reference. Table No. 7 is printed with this discussion (see pages 468 and 469) in order to show the comparison referred to.